

In a Nutshell

Everything you always wanted to know about ...

Accuracy, Precision, Uncertainty, and Significant Figures.

Accuracy describes reliability as to agreement with fact, standard or true value. Precision describes repeatability including random scatter of measured values.

SUMMARY:

1. Take a **single** measurement and combine into one total uncertainty, the manufacturer's accuracy, reading uncertainty, and the operator uncertainty.
2. Take several identical, independent measurements and obtain the **average**. Simplest: determine the precision uncertainty as half the range.
Range \equiv Maximum possible value - Minimum possible value.
3. Plot a graph of a set of measured data points and graphically **average** over the measured data points using a best-fit line model with its scatter band to represent the correlation between the measured variables.
4. *Always report-out* three bits of information: value, uncertainty, and unit(s) in order to represent the result of any quantitative measurement.
5. Ensure the uncertainty (always 1 digit) talks about the right-most column.
Rule: The first column whose entry is in doubt, is the last one given out. or **The last column given out is in doubt.** This rule always applies: including measurements, textbook calculations, and theory calculations.
6. The number of **significant** figures is the number of columns that are filled in when the amount of a quantity is written in standard scientific notation.
7. **Round off** a number *only* when giving it out to someone else; thus convey to the receiver how far a number can be trusted (via its significant figures).
8. Uncertainty always indicates both the **amount** of doubt (the single digit) and the doubt **column** it refers to (the last or right-most column in the value).
Examples:
 - ± 2 s refers to the seconds (unit) column
 - ± 0.002 s (± 2 ms) refers to the millisecond column
 - $\pm 0.000\ 002$ s (± 2 μ s) refers to the microsecond column

Measurement is a comparison of an unknown quantity with a well-defined standard or reference quantity called the *standard* or the reference.

All measurements have doubt or uncertainty associated with them; measuring instruments and our ability to read and operate them will always be with us. A number of terms are often used in conjunction with measurements in order to describe the exactness or preciseness of the measured values.

- Accuracy describes agreement with fact, standard, or truth. It represents the ability of an instrument to provide correct values, assuming the user is able to operate and read it perfectly. The manufacturer of the instrument normally specifies the accuracy of an instrument.
- Precision describes sharpness of definition. It is normally related to the gradations on the measuring instrument and the ability of the user to interpret between those gradations, as well as the ability of the user to properly operate the instrument or perform the measurement.
- Uncertainty describes the range within which a true or correct value has a specified probability of being found; it combines both accuracy and precision.
- Tolerance is the allowable deviation from a specified value. The designer or engineer normally specifies the tolerance in order to provide limitations on the design value that will be acceptable and still enable the designed system to function.
- Discrepancy is the difference between two values - measured and another value. It may be expressed as absolute (having the units of the values) or as relative to some value, preferably the “true or best known” value, (for example as a percent).
- Error is the acknowledged discrepancy between a measured value and the true, correct, or best value. It is usually expressed as a positive value and as a percentage of the “true or correct” value.

Note:

All of the above terms are used to indicate an amount of *doubt* in one particular column and therefore can be only one single digit, not greater than nine units in the appropriate last right-most column of the value; the column whose entry is in doubt.

In Practice:

When recording or communicating a number, start by filling in columns with digits (the first column is always a non-zero digit) and continue until one encounters a column whose entry is in doubt, which is the last cut-off column. The number of columns thus filled in is referred to as the number of significant figures. Thus always this convention:

The first column whose entry is in doubt is the last one given out (and vice versa).

Note

- 1 It is senseless to go to a column beyond the last doubtful column as its units are (at least) ten times smaller and therefore its amount will most certainly be unknown.
- 2 The amount of doubt cannot be greater than nine units in the appropriate last cutoff column, that is the column containing the doubtful digit or the doubtful column.
- 3 The decimal point (if any) may be shifted by adjusting the power of ten multiplier. Decimal means “fraction to follow” thus an integral number, having no need for a decimal point, is said to have infinite or unlimited significant figures. (example: forty-two 42 persons in a room).
- 4 If necessary, round off to conform to this conventional rule. One rounds off a number in order to convey to the recipient (via the number’s significant figures) how far the number can be trusted.

Precision describes the sharpness of definition.

example:

Suppose the width of a book is found to be no less than 184 mm and no greater than 189 mm. The possible width is thus anywhere from 184 to 189 mm or its **range** is 5 mm which is a measure of its sharpness of definition. The book’s width would be communicated as 187 ± 3 mm or 187(3) mm, where the ± 3 mm represents the precision of the book’s width. Note the rounding to the appropriate doubtful column and never pretend to know a precision to more than one single digit.

example:

Suppose the ratio of circumference to the diameter of a circle is given to be 3.1412
The implied amount of doubt or precision cannot be greater than ± 0.0009 , in fact it may be less.

Accuracy describes agreement with fact, standard, or truth.

example:

Suppose in the above book example the divisions on the ruler were all made too small by the manufacturer of the ruler. Then no matter how well the ruler is read or operated, one will never obtain the true value of the book’s width using this particular ruler. The system used to measure the width, this ruler, has an inherent (built-in) discrepancy which results in an **error** compared to the truth. If with a different, more perfect ruler the width was determined to be 179.365(4) mm or its true width is somehow known to be say 179.365 mm, then the book’s width of 187(3) mm is inaccurate by 8 mm (187 - 179.365), again to only one digit..

example:

In the above example of the ratio of circumference to the diameter of a circle, the true value is known to be 3.14159265... Hence, the above quoted value of 3.1412 is accurate to 0.0004.

Scientific Notation

Standard or scientific notation is used to abbreviate large numbers and to avoid ambiguity in significant figures. To express a number in scientific notation, the number is written with **only one non-zero digit placed before the decimal point**, followed by the appropriate power of ten. The number of significant figures is the number of filled-in columns.

i.e. number = n.nn x 10^m example Number = 7.16 x 10³

The “n.nn” or 7.16 is the ***mantissa*** and “m” or 3 is the ***power*** or ***order of magnitude***.

The number is thus written with only one non-zero digit placed before the decimal point. All additionally filled-in columns are also meaningful or significant because no extra zeros are used to ‘hold’ a decimal place. Significant figures in any number are thus most easily indicated using this standard notation, plus metric prefixes if desired.

Example: (8.321 ± 0.006) x 10⁶ = (8.321 ± 0.006) Mega (M) has 4 sig fig
 (1.40 ± 0.05) x 10⁻⁶ = (1.40 ± 0.05) micro (μ) has 3 sig fig

1. Addition or subtraction

First write the numbers in ordinary form or all with the same power of ten. After the calculation round off to the first column that is in doubt.

Example:

3.1416 x 10 ¹	31.416	3.146 x 10 ¹
+ 8.203 x 10 ³	+ 8203	+ 820.3 x 10 ¹
	= 8234	= 823.4 x 10 ¹

Shortcut: The total uncertainty is the sum of the individual ***absolute*** uncertainties. (see page 9)

2. Multiplication or Division

For convenience, separate the mantissas and the powers and calculate separately. After the calculation round off to the first column that is in doubt.

Example:

$$\frac{8.32 M}{0.31 m} = \frac{8.32 \times 10^6}{3.1 \times 10^{-4}}$$

$$= \left(\frac{8.32}{3.1} \right) \times 10^{+6-(-4)}$$

$$= 2.6838 \times 10^{10} \quad \left(\frac{\text{maximum possible range} = 1.76\dots}{2} = \pm 0.9 \right)$$

$$= 27 \text{ Giga (two significant figures)}$$

Shortcut: The total uncertainty is the sum of the individual ***relative*** uncertainties. (see page 9)

Metric Prefixes

Prefix Name	Symbol	Definition		Equivalent
exa	E	10^{18}	1 with 18 zeros	
peta	P	10^{15}	1 with 15 zeros	
tera	T	10^{12}	1 000 000 000 000	trillion
giga	G	10^9	1 000 000 000	billion
mega	M	10^6	1 000 000	million
kilo	K	10^3	1 000	thousand
hecto	h	10^2	100	hundred
deca	da	10^1	10	ten
		1		one
deci	d	10^{-1}	0.1	tenth
centi	c	10^{-2}	0.01	hundredth
milli	m	10^{-3}	0.001	thousandth
micro	μ	10^{-6}	0.000 001	millionth
nano	n	10^{-9}	0.000 000 001	billionth
pico	p	10^{-12}	0.000 000 000 001	trillionth
femto	f	10^{-15}	1 in 15 th dec place	
atto	a	10^{-18}	1 in 18 th dec place	

The more frequently used prefixes are indicated in **bold** (heavy type) in the table.

Significant Figures

We record or give out only meaningful or significant digits. The convention is as follows: the rightmost (last) column contains the doubtful digit; the digit that has the uncertainty. Thus a measurement's doubt or uncertainty **must** talk about the rightmost (last) column.

The first column whose entry is in doubt is the last one given out.

This concept of significant figures provides a simple and convenient way of implying the maximum possible uncertainty in a value; it is at most 9 units in the last column given out. For example, an electrical power plant producing say 8.43 MegaWatt of power **implies** an uncertainty no greater than ± 0.09 MW, in fact its uncertainty may be as little as ± 0.01 .

Rounding Off

Rounding off refers to how well a value can be trusted; the rightmost column is in doubt. How much doubt in that last, right-most column? at most nine (± 9) units! In order to round off a calculated result, the last significant digit (right-most column) is increased by one if the digit following it is five (5) or greater, otherwise it remains the same. In either case the digits following the last significant column are dropped.

Example: $143.66 + 4.1 + 0.184 = 147.944 \dots$ rounds off to 147.9
 $45.50 \div 5.30 = 8.5849\dots$ rounds off to 8.58

A word of **caution**, in order to avoid round-off error propagation **never** round off until the final answer is obtained after all the necessary operations have been carried out and you are ready to give out the (rounded-off) result to someone else. The recipient can then tell how far the result can be trusted, using the above concept of significant figures.

What Happens to the Sig Figs when I Add / Subtract / Multiply / Divide Numbers?

1. Add / Subtract:

Round off the answer to its first column entry from the left that contains any doubt.

2. Multiply / Divide

Round off the answer to the same number of significant figures as the number with the least number of significant figures in the calculation. This quick shortcut for calculations will give the correct number of significant figures in the answer most of the time.

Exercises

1. The diameter of a circle is measured to be 6.072 ± 0.005 cm. Calculate the area of the circle and its error assuming that $\pi = 3.14151926\dots$ Answer: 28.96 ± 0.05 cm²
2. The distance an object travels and the time taken are measured to be (1.748 ± 0.009) cm and $(5.41 \pm 0.05) \times 10^{-3}$ seconds. Calculate the speed. Answer: 3.23 ± 0.05 m/s

Significant Figures - Exercises

Answers to the following questions are given in the table below.

1. State the number of significant figures in each of the following.

- | | | | | | |
|-----------------------|-------|------------|-------|------------------|-------|
| a) 256 | _____ | b) 2560 | _____ | c) 2.56 | _____ |
| d) 2.560 | _____ | e) 0.2560 | _____ | f) 0.0256 | _____ |
| g) 0.00001 | _____ | h) 10007 | _____ | i) 2000 ± 10 | _____ |
| j) 2.00×10^3 | _____ | k) 0.05800 | _____ | l) 302 | _____ |

2. Round off the following to two significant figures.

- | | | | | | |
|-----------------------|-------|------------|-------|-------------|-------|
| a) 343 | _____ | b) 348 | _____ | c) 345 | _____ |
| d) 598 | _____ | e) 6045 | _____ | f) 6055 | _____ |
| g) 901 | _____ | h) 0.0121 | _____ | i) 0.03029 | _____ |
| j) 2.75×10^3 | _____ | k) 0.06300 | _____ | l) 0.008949 | _____ |

3. Perform the arithmetic and round off the result to the correct number of significant figures.

$$5610.04 + 8.5 + 765 = \underline{\hspace{10em}} = 6.384 \times 10^3$$

$$\frac{73215}{5} = \underline{\hspace{10em}} = 1 \times 10^4$$

$$83 \times 25.0 \times 1.2 = \underline{\hspace{10em}} = 2.5 \times 10^3$$

$$75 \times 0.500 = \underline{\hspace{10em}} = 3.8 \times 10^1$$

4. Express the following numbers in correct exponential notation. Use prefixes where appropriate.

- | | | | | | |
|-----------|-------|------------|-------|-----------------------|-------|
| a) 1000 | _____ | b) 10.00 | _____ | c) 1000 ± 20 | _____ |
| d) 0.0001 | _____ | e) 0.00010 | _____ | f) 2568 | _____ |
| g) 0.0256 | _____ | h) 2.56 | _____ | i) 18.3×10^3 | _____ |
| j) 7380 | _____ | k) 738.0 | _____ | l) 7380 ± 1 | _____ |

5. Perform the arithmetic and answer in exponential notation with the correct significant figures.

$$(2.5 \times 10^2) + (3.8 \times 10) + 765 = \underline{\hspace{10em}} = 1.05 \times 10^3$$

$$6.83 \times 10^{-8} \times 7.1 \times 10^7 = \underline{\hspace{10em}} = 4.8$$

$$1.50 \times 10^{10} \times 3.2 \times 10^8 = \underline{\hspace{10em}} = 4.8 \times 10^{18}$$

$$\left(\frac{3.6 + 5}{1.2 \times 10^3} \right)^2 = \underline{\hspace{10em}} = 5 \times 10^{-5}$$

Answers: E3 ≡ Kilo E-3 ≡ milli

	a)	b)	c)	d)	e)	f)	g)	h)	i)	j)	k)	l)
1	3	3	3	4	4	3	1	5	3	3	4	3
2	34 E1	35 E1	0.35 E3	6.0 E2	6.0 E3	6.1 E3	9.0 E2	1.2 E-2	3.0 E-2	2.8 E3	6.3 E-2	8.9 E-3
4	1 E3	1.000 E1	1.00 E3	1 E-4	1.0 E-4	2.568 E3	2.56 E-2	2.56	1.83 E4	7.38 E3	7.380 E2	7.380 E3

Uncertainty

Uncertainty describes the range within which a true or correct value has a specified probability of being found; it combines both accuracy and precision.

The accuracy Δa and the precision Δp both affect the uncertainty. Statistically, they are usually combined 'pythagoras fashion' into the total uncertainty:

$$\text{uncertainty} = \sqrt{(\Delta a)^2 + (\Delta p)^2} = \sqrt{(0.003 \text{ mm})^2 + (0.005 \text{ mm})^2} = \pm 0.006 \text{ mm}$$

Where the numbers refer to an example when a technician measures the diameter of a cylinder as 3.580 mm with a precision of ± 0.005 mm (or 5 μm) using an instrument that has accuracy of 0.003 mm (or 3 μm).

Therefore, the measured value is 3.580 ± 0.006 mm or ± 6 μm . This means the true value is probably between 3.574 and 3.586 mm.

Example:

A student is asked to determine the area of a tabletop. He measures the width of the table to be 52.3 ± 0.2 cm and its length as 85.7 ± 0.3 cm. What is the area of this tabletop?

$$W = 52.3 \pm 0.2 \text{ cm} = 52.3 \text{ cm} \pm 0.4 \%$$

$$L = 85.7 \pm 0.3 \text{ cm} = 85.7 \text{ cm} \pm 0.4 \%$$

$$\text{Area } A = L \times W = (52.3 \pm 0.2 \text{ cm}) \times (85.7 \pm 0.3 \text{ cm})$$

$$\% A = \sqrt{(\% L)^2 + (\% W)^2} = \sqrt{(0.4 \%)^2 + (0.4 \%)^2} = 0.6 \%$$

$$\text{Area } A = 4482.11 \text{ cm}^2 \pm 0.6 \% = (4482.11 \pm 27) \text{ cm}^2$$

The area of the tabletop is reported, correctly rounded, (with or without 'pythagoras fashion' combination) as:

$$\begin{aligned} \text{Area} &= (448 \pm 3) \times 10^1 \text{ cm}^2 \\ \text{or Area} &= 448 \times 10^1 \text{ cm}^2 \pm 0.7 \% \end{aligned}$$

Example: using or calculating extreme values *always* works:

$$\begin{array}{l} \text{maximum } \cos(8^\circ) = 0.990 \\ \cos(10 \pm 2^\circ) = \cos(10^\circ) = 0.985 \quad \text{Answer is: } 0.985 \pm 0.006 \\ \text{minimum } \cos(12^\circ) = 0.978 \end{array}$$

where we used *the uncertainty is half the range*.

Example: the uncertainty talks about or is linked to the last, right-most column:

$$\begin{array}{llll} 69.4 \text{ cm} & 143.07 \text{ kg} & 15.0 \text{ second} & 0.985 \\ \pm 0.3 \text{ cm} & \pm 0.06 \text{ kg} & \pm 0.2 \text{ second} & \pm 0.006 \end{array}$$

Another way of reporting $143.07 \pm 0.06 \text{ kg}$ is **143.07(6) kg** indicating an uncertainty of 6 in the last or right-most column.

Example: we subtract two distances to obtain a displacement:

$$\begin{array}{llll} \text{if} & x_2 = & 143.07 & \pm 0.06 \text{ cm} \\ \text{and} & x_1 = & 4.1 & \pm 0.7 \text{ cm} \\ \text{then the change in } x \text{ is } \Delta x = x_2 - x_1 = & 139.0 & \pm 0.8 & \text{ cm} \end{array}$$

where we added **absolute** uncertainties

Shortcut: The total uncertainty is the sum of individual **absolute** uncertainties.

Example: we multiply for the power dissipated in an electrical resistor:

$$\begin{array}{ll} \text{if current } I = & 70.0 \pm 0.3 \text{ mA} \\ \text{and resistance } R = & 48.0 \pm 0.5 \text{ ohm} \end{array}$$

then the power dissipated is:

$$\begin{aligned} \text{Power } P &= I^2 R = (70.0 \text{ mA} \pm 0.4\%) (70.0 \text{ mA} \pm 0.4\%) (48.0 \pm 1\%) \\ &= 235 \text{ mW} \pm 2\% \end{aligned}$$

where we added **relative** uncertainties

Shortcut: The total uncertainty is the sum of individual **relative** uncertainties.

Example: we propagate uncertainty by converting between absolute and relative uncertainty as necessary.

$$\text{density} \equiv \frac{\text{mass of a sphere}}{\text{volume of a sphere}}$$

$$= \frac{73.2 \pm 0.3 \text{ kg}}{\frac{4}{3} \pi \left\{ (9.34 \pm 0.01 \text{ m}) + (4.21 \text{ m} \pm 2\%) \right\}^3}$$

convert to absolute uncertainty

$$= \frac{73.2 \pm 0.3 \text{ kg}}{\frac{4}{3} \pi \left\{ (9.34 \pm 0.01 \text{ m}) + (4.21 \text{ m} \pm 0.0842 \text{ m}) \right\}^3}$$

absolute uncertainty path

relative uncertainty path

$$= \frac{73.2 \pm 0.3 \text{ kg}}{\frac{4}{3} \pi \left\{ 13.55 \pm 0.0942 \text{ m} \right\}^3}$$

$$= \frac{73.2 \text{ kg} \pm 0.4\%}{\frac{4}{3} \pi \left\{ 13.55 \text{ m} \pm 0.7\% \right\}^3}$$

$$= \frac{73.2 \pm 0.3 \text{ kg}}{10,421 \pm 219 \text{ m}^3}$$

$$= \frac{73.2 \text{ kg} \pm 0.4\%}{10,421 \text{ m}^3 \pm 2.1\%}$$

$$= 7.0 \pm 0.2 \frac{\text{gram}}{\text{m}^3}$$

$$= 7.0 \frac{\text{gram}}{\text{m}^3} \pm 3\%$$

$$\text{or} = 7.0(2) \frac{\text{gram}}{\text{m}^3}$$

The reader should verify each path by itself, and thereby verify that each path (absolute or relative) does indeed lead to the same result.

Comparisons

It is often useful to compare two values of the same physical quantity. Do two values agree?

Difference Discrepancy Deviation Error

The difference between an experimentally measured value M and some other value V is:

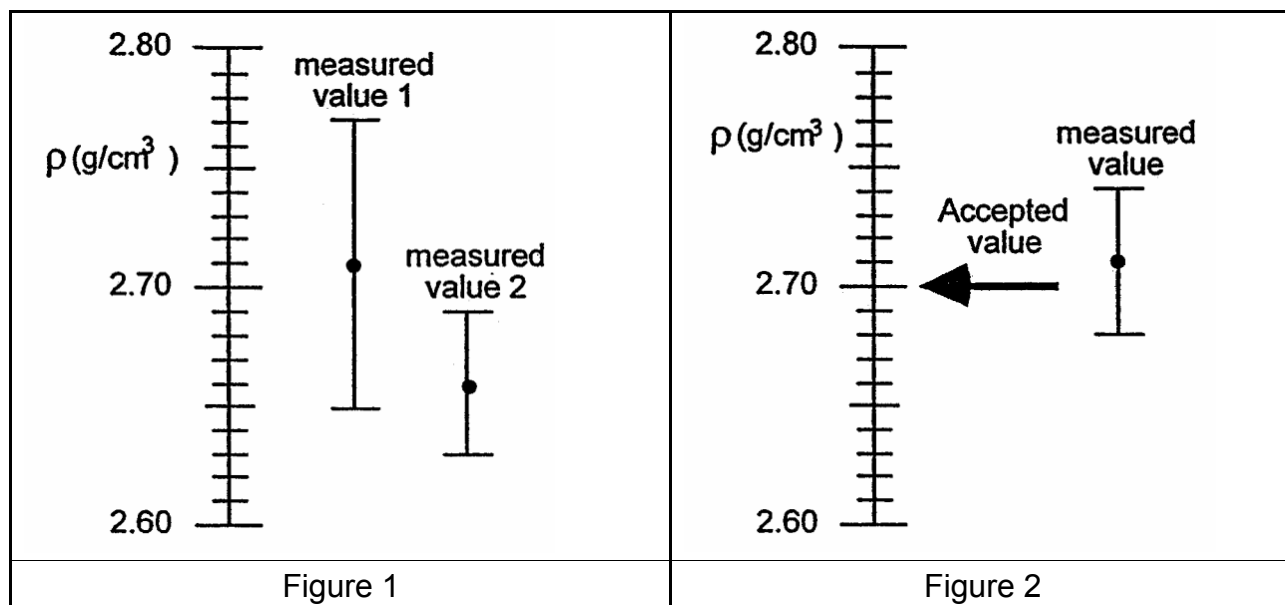
$$\text{difference} = M - V \quad \text{or} \quad \% \text{ difference} = \frac{M - V}{V} \times 100\%$$

Note that the **sign** of the difference will thus indicate whether the measured value is larger (positive) or smaller (negative) than the other value. The value V is usually meant to be the 'true' value or the design value. If the true value is not known one substitutes for V the public published value also known as the **accepted** value. If the 'true' value of a quantity is known then one speaks of or admits to **error** instead of difference or discrepancy.

Measured quantities are always written in the form of **value \pm uncertainty** with units. For example, the density of aluminum may be measured to be $2.70 \pm 0.04 \text{ gram/cm}^3$ or $2.70(4) \text{ gram/cm}^3$. By this we mean that we believe the true (and ultimately unknowable density) of aluminum to be somewhere in the range 2.66 to 2.74 gram/cm^3 .

The comparison between two or more values may conveniently be done graphically by displaying the values along with their uncertainties. We will do so for two basic types.

The first (Figure 1) compares two or more measured quantities with each other, while the second (Figure 2) compares one or more measured quantities with a widely accepted value. The accepted value may also be a measured value in which case it should have its own uncertainty range. However, this uncertainty is usually so small compared to the uncertainty of our own measurement that it does not show up on our comparison graph.



In Figure 1 the range for the value 1 is $2.65\text{--}2.77 \text{ gram/cm}^3$ and the range for the value 2 is $2.63\text{--}2.69 \text{ gram/cm}^3$. These two measured values **share the range** $2.65\text{--}2.69 \text{ gram/cm}^3$. Because they have a range of common values (their ranges overlap), we say that value 1 and value 2 are **experimentally equal**. In Figure 2 the accepted value lies inside the range of the measured value, so they are experimentally equal.